

Journal of Financial and Actuarial Mathematics and Management

A joint edition of the Institute of Information Sciences and Institute of Social Sciences of Perun Academy
Publishing Company: PSP Publishing
ISSN: 2367-5810
Issue 13, Year XIII (2025)
Founding Editor and Editor-in-Chief: Pavel Stoynov

Editorial Board: Hiroyuki Ono (Japan), Yasukazu Yoshizawa (Japan), Naoyuki Ishimura (Japan), Boris Melnikov (Russia), Elena Melnikova (Russia), Goran Karanovic (Croatia), James Osondu (Nigeria)

Contents

P. Stoynov Creating an international financial group –
a case study with Perun Finance Holding (PFH)…….....3
Y. Abd Algani. Zermelo-Fraenkel set theory (ZF) –
a study on the essential axioms for a better application
of the foundational theory.....16
P. Stoynov. Kumaraswamy-Switch-Time distributions.....24

Creating an international financial group – a case study with Perun Finance Holding (PFH)

Pavel Stoynov
Perun R&D Acadaemy
Institute of Technologies,
Sofia, Bulgaria
pavel_todorov_stoynov@yahoo.com

"The best time to plant a tree was 20 years ago. The second best time is now." - Chinese Proverb

Abstract.
In this article we consider some issues related to creating a new international financial group using a specific case – prospective creation of Perun Finance Holding (PFH).
Keywords: finance, financial group, funding, business plan,

Introduction

Perun Finance is a prospective global financial group containing: a bank, an insurance company, a pension fund, and an investment fund. It will be structured as a Financial Holding Company (Perun Finance Holding - PFH). This structure allows for unified strategic oversight, shared support functions (like IT and HR), and legal separation of high-risk activities, ensuring regulatory compliance across multiple jurisdictions and distinct financial sectors (banking, insurance, asset management).

Entities of Perun Finance Holding and their activities

The financial group structure includes the following entities:

1. Perun Finance Holding (PFH). It will be the holding company, the central of the all group. The main its activities will be: central oversight, group strategy, capital allocation, Group Compliance & Risk Management, M&A. It must comply with the requirements of the home country banking supervision. For EU countries, like the planned head quarter – Bulgaria, EU – these wil be the rules of ECB (2026).
2. Perun Bank (PEB). It is the bank of the financial group with universal banking activities: retail, commercial, and investment banking services worldwide. For EU countries it should comply to Basel III (2026).
3. Perun Assurance (PEA). It is the insurance company in the group. Its main activities are Insurance & Reinsurance: Life, Non-Life, and Health coverage. It should comply to Solvency II (EU) and equivalent global regulations
4. Perun Pensions (PEP). It is the pension fund of the group and performs pension fund management activities: mandatory and voluntary schemes; long-term, low-risk investment focus. The regulatory rules are from IORP II (EU) and country-specific pension regulations.
5. Perun Capital (PEC). It is the investment fund of the group and performs conventional and alternative Investments. The regulation is according to AIFMD (EU) and SEC/FCA regulations (US/UK).

In some markets, the bank, the assurance company and the investment fund may be replaced by financial brocker and insurance broker.

The operational entities (Bank, Insurance, Pension, Investment Fund) would use a mixed structure, including primary hubs in the international financial centers (London, Paris, Frankurt, New York, Singapore, Hong Kong) local subsidiaries (fully capitalized, locally regulated subsidiaries or branches in major markets) and representative offices (smaller presence for business development in markets like Nigeria, Australia, Brazil).

Perun Bank (PEB) has as primary function to provide deposit-taking, lending, payments, and capital markets services.

Departments and main processes:

1. Retail Banking

Deposits: Account opening/maintenance, KYC/AML check.
Lending: Mortgage/Consumer loan origination, credit scoring, CRM.
Channels: Branch network management, mobile/online banking UX.

2. Corporate & Commercial Banking

Corporate Lending: Credit analysis, term loans, trade finance (Letters of Credit).
Treasury: Cash management services, FX hedging for corporate clients.

3. Investment Banking

M&A Advisory: Deal sourcing, valuation, due diligence, transaction execution.

Capital Markets: IPO structuring, debt issuance (DCM), equity placement (ECM).

4. Treasury & ALM (Asset-Liability Management)

Funding: Interbank borrowing/lending, bond issuance.

Liquidity: Managing LCR and NSFR ratios.

Risk: Interest rate risk management, balance sheet optimization.

Credit Risk Underwriting: Setting credit policies, approving large loans/limits. Monitoring: Portfolio stress testing, bad loan provisioning.

Perun Assurance (PEA) has as a primary function to underwrite and manage insurance risk (Life and Non-Life).

Its departments are:

1. Actuarial & Product Development

Pricing: Calculating premiums based on probability (frequency x severity).

Reserving: Calculating technical provisions (Best Estimate Liabilities) under Solvency II.

Product: Designing new insurance policies (e.g., cyber, climate risk).

2. Underwriting

Risk Assessment: Evaluating individual or corporate risk applications.

Acceptance/Rejection: Deciding on policy issuance, setting specific terms and deductibles.

3. Claims Management

Claim Intake: Receiving, validating, and registering claims (FNOL).

Investigation: Claims adjustment, fraud detection.

Settlement: Authorizing and executing claim payments.

4. Investments

Asset Management: Investing the float (premiums received but not yet paid out as claims) to match long-term liabilities.

ALM: Coordinating with Actuarial to manage asset-liability mismatch risk.

5. Reinsurance & Retrocession

Risk Transfer: Ceding catastrophic or large-volume risks to global reinsurers.

Reinsurance Accounting: Managing cash flows with reinsurers.

Perun Pensions (PEP) has as a primary function to manage pension assets and liabilities for long-term secure returns.

The main departments are:

1. Fund Administration & Operations

Record Keeping: Managing individual member accounts, contribution tracking, benefit payments.

Reporting: Regulatory reporting to pension supervisors.

2. Investment Management

Strategic A/A: Setting long-term asset allocation strategy (e.g., 60% Bonds, 30% Equity, 10% Alternatives).

Execution: Buying/selling assets to rebalance the portfolio.

3. Member Services & Communication

Onboarding: Enrollment of new members.

Support: Handling member queries (e.g., retirement projections, contribution history).

Disclosure: Providing annual statements and regulatory disclosures.

4. Actuarial & Liability Modeling

Valuation: Calculating the Present Value of future pension obligations.

Solvency: Ensuring the fund is adequately capitalized to meet future payments.

Perun Capital (PEC) has as a primary function to perform asset management (Fund activities).

The main departments are:

1. Portfolio Management (Front Office)

Research: Idea generation, fundamental/quantitative analysis, model building.

Trading: Order placement, best execution, intra-day risk management.

Strategy: Implementing specific strategies (Long/Short Equity, Global Macro, Arbitrage).

2. Risk Management (Middle Office)

VaR Calculation: Measuring market risk (Value at Risk).

Exposure Monitoring: Tracking leverage, concentration limits, and counterparty credit risk.

Stress Testing: Simulating portfolio performance under adverse scenarios.

3. Operations & Settlement (Back Office)

Trade Processing: Recording and confirming all executed trades.

Settlement: Ensuring accurate cash and security transfers.

Reconciliation: Matching internal records with custodian/broker statements.

4. Investor Relations & Marketing

Fundraising: Pitching strategies to Qualified Investors (Endowments, SWFs).

Reporting: Providing detailed performance and risk reports to investors.

Perun Finance Holding (PFH). It performs group level functions (Shared Services under PFH).

Its departments operate at the Holding Company (PFH) level to provide central control and efficiency across all four entities.

The main departments are:

1. Group Compliance & AML

Policy: Setting group-wide compliance standards (GDPR, MiFID II, FATCA).

Monitoring: Auditing subsidiaries for adherence to AML and KYC rules.

Regulatory Relations: Coordinating with global regulators (ECB, SEC, PRA).

2. Group Finance & Treasury

Consolidation: Preparing consolidated financial statements (IFRS).

Tax: Managing global tax structures and compliance.

Capital Allocation: Optimizing regulatory capital distribution across entities.

3. Group IT & Cybersecurity

Infrastructure: Managing shared data centers, cloud infrastructure, and network security.

Development: Core Banking System (CBS) and ERP maintenance, FinTech integration.

4. Group HR & Training

Talent Management: Global recruitment, compensation, and succession planning.

Learning & Development: Mandatory compliance training, leadership programs.

Estimated Capital Requirement

The capital needed to run a successful, globally diversified financial group of this scale (presence in 12-16 countries) is substantial, driven primarily by the regulated entities.

For banks many countries adhere to the international standard for banking supervision, **Basel III**, developed by the Basel Committee on Banking Supervision (BCBS). Basel III establishes global standards for bank capital adequacy, stress testing, and market liquidity risk.

The requirements are:

Common Equity Tier 1 (CET1) Ratio should be greater thamn or equal to 4.5% of Risk-Weighted Assets (RWA).

Tier 1 Capital Ratio should be greater than or equal to 6.0% of RWA (CET1 + Additional Tier 1).

Total Capital Ratio should be greater than or equal to 8.0% of RWA (Tier 1 + Tier 2 Capital).

Additional 2.5% Capital Conservation Buffer buffer is required on top of the ratios (must be CET1).

A variable Countercyclical Capital Buffer (CCyB) of 0-2.5% is based on national credit cycles.

Additional Higher Loss Absorbency (HLA) requirement is applied to Global Systemically Important Banks (G-SIBs).

The minimum ratios stated above are the international floor; many countries impose higher domestic requirements. The specific requirements are often complex, subject to ongoing revisions, and tailored by each country's financial regulator.

Below we provide some specific cases in selected countries.

European Union Members (Bulgaria, Germany, France).

As EU member states, Bulgaria, Germany, and France are subject to the EU's financial regulations, primarily the Capital Requirements Regulation (CRR) and Capital Requirements Directive (CRD), which implement the Basel III framework.

France & Germany: Are subject to EU-level and national supervision (ECB/SSM for large banks).

Bulgaria: Banks in Bulgaria have historically maintained a very high average capital adequacy ratio (e.g., around 22.7% as of 2025, well above the EU average) for stability.

United Kingdom (UK). The UK, regulated by the Prudential Regulation Authority (PRA) and Financial Conduct Authority (FCA), has implemented its own version of Basel III post-Brexit, maintaining close alignment with international standards.

Capital Ratios: Follows the Basel III minimums (8.0% Total Capital, 4.5% CET1) plus the 2.5% Capital Conservation Buffer and the UK's specific CCyB.

Ring-fencing: Larger UK banks must legally separate their essential retail banking from their investment banking activities.

United States (USA). Regulators include the Federal Reserve (Fed), Office of the Comptroller of the Currency (OCC), and Federal Deposit Insurance Corporation (FDIC).

Capital Ratios: US banks must meet the Basel III requirements, often called the 'Standardized Approach', and may also be subject to the 'Advanced Approaches' for calculating RWA. They have specific categories like "well-capitalized" (higher than minimum requirements).

Canada. Regulated by the Office of the Superintendent of Financial Institutions (OSFI).

Capital Ratios: Canadian banks follow the Basel III framework. OSFI typically sets a higher supervisory target capital ratio (Total Capital Target: 10.25%) than the Basel III minimum.

Russia. Regulated by the Central Bank of the Russian Federation (CBR).

Capital Ratios: The CBR also adheres to Basel III principles, setting minimum capital adequacy ratios for banks. The required reserve ratio is a key monetary policy tool.

Turkey. Regulated by the Banking Regulation and Supervision Agency (BRSA).

Capital Ratios: Turkey has been incorporating Basel III rules, including an indicator-based approach to identify Domestic Systemically Important Banks (D-SIBs) which require higher loss absorbency buffers.

Minimum Share Capital: Specific regulations govern the minimum share capital for various financial institutions, which are separate from RWA-based capital adequacy ratios.

China. Regulated by the China Banking and Insurance Regulatory Commission (CBIRC).

Capital Ratios: China largely follows the Basel III framework. Large banks, especially Globally Systemically Important Banks (G-SIBs), face additional capital requirements, such as a higher CET1 ratio floor.

India. Regulated by the Reserve Bank of India (RBI).

Capital Ratios: The RBI implements Basel III. Banks are required to maintain a higher Total Capital Ratio of 11.5% (vs. the 8% Basel minimum), including the Capital Conservation Buffer.

Specific Banks: Specialized banks like Small Finance Banks have a specific minimum initial net worth requirement.

Japan. Regulated by the Financial Services Agency (FSA) and the Bank of Japan (BOJ).

Capital Ratios: Adheres to Basel III. Banks with international operations must meet the standards, while domestic-only banks have slightly different rules. The minimum stated capital for a new bank is ¥2 Billion.

Liquidity: The FSA has implemented the Basel III Liquidity Coverage Ratio (LCR) requirements.

Brazil. Regulated by the Central Bank of Brazil.

Capital Ratios: Implements the Basel III framework, including a focus on Total Capital, Tier 1, and CET1 ratios, along with the capital conservation buffer. Requirements have been adjusted to also cover non-traditional financial institutions like large fintechs/payment institutions.

The regulatory requirements for insurance are comprehensive, multi-layered, and designed primarily to protect policyholders and ensure the financial stability of the insurance market. These requirements generally fall into three key categories, often referred to as "Pillars" in many modern regimes like the **EU's Solvency II**:

Pillar 1: Financial Requirements (Capital and Solvency)

This is the most critical pillar, focusing on the insurer's ability to cover its risks and meet its financial obligations.

Minimum Capital Requirement (MCR): The absolute minimum amount of capital an insurer must hold to operate. Falling below this level usually triggers immediate and serious supervisory intervention, potentially leading to the loss of authorization.

Solvency Capital Requirement (SCR) / Risk-Based Capital (RBC): This is a risk-based capital target, generally higher than the MCR, calibrated to ensure the insurer can absorb significant unforeseen losses (e.g., up to a 1-in-200-year event, as in Solvency II).

In the EU/UK, this is defined by Solvency II / Solvency UK, which uses a Standard Formula or an approved Internal Model to calculate the SCR, covering risks like underwriting, market, credit, and operational risk.

In the US, the National Association of Insurance Commissioners (NAIC) sets the framework for state-level Risk-Based Capital (RBC) requirements, linking required capital to the insurer's specific risks.

Technical Provisions (or Insurance Liabilities): Requirements for how an insurer must calculate and reserve for its future payment obligations to policyholders. Modern regimes, like Solvency II and those evolving in Japan and other countries, require an "economic value-based" or "market-consistent" valuation of these liabilities, including a Best Estimate and a Risk Margin.

Admissible Assets and Investment Rules: Rules that govern the quality and diversification of assets an insurer holds to back its technical provisions. While modern rules often follow a 'Prudent Person Principle', requiring insurers to invest responsibly, some jurisdictions may still have prescriptive limits on asset classes.

Pillar 2: Governance, Risk Management, and Supervision

This pillar ensures the insurer has sound internal systems and controls to manage its business and risks.

System of Governance: Requirements for a robust organizational structure, clear allocation of responsibilities, and effective internal controls.

Fit and Proper Requirements: Rules ensuring that directors and senior managers possess the competence, integrity, and financial soundness necessary to manage the company.

Key Functions: Mandatory appointment and effective operation of specific independent functions, typically including:

Actuarial Function: Responsible for calculating technical provisions and ensuring the overall solvency system is sound.

Risk Management Function: Responsible for implementing the risk management system and monitoring the overall risk profile.

Compliance Function: Responsible for advising the board and management on compliance with laws and regulations.

Internal Audit Function: Provides independent and objective assurance on the effectiveness of the governance and control systems.

Own Risk and Solvency Assessment (ORSA): A crucial requirement in many regimes (e.g., EU, Brazil, Canada) where the insurer must conduct its own assessment of its overall solvency needs, considering its specific risk profile and future strategy, and demonstrate that its capital is adequate for its risks.

Pillar 3: Reporting and Transparency (Public Disclosure)

This pillar ensures that regulators and the public receive adequate, timely, and comparable information about the insurer's financial condition and risk profile.

Regulatory Reporting: Detailed and regular reporting of financial and non-financial data to the supervisory authority (e.g., quarterly, annual reports on assets, liabilities, capital, and risk exposure).

Public Disclosure: Requirements for making certain information publicly available to enhance market discipline and transparency. This typically includes a Solvency and Financial Condition Report (SFCR) or similar document detailing the insurer's business, governance structure, risk profile, and capital management.

Market Conduct: Rules governing how insurers interact with customers, including requirements for clear and fair marketing, sales practices, product suitability, and efficient, transparent claims settlement procedures.

Anti-Money Laundering (AML) / Counter-Terrorist Financing (CTF): Obligations to implement systems to detect and prevent financial crime.

In essence, these regulations ensure that insurers are safe (financially sound), fair (treat customers ethically), and transparent (provide clear information to regulators and the public).

Some specificities for insurance regulation in selected countries are given below.

Insurance regulation in **France and Germany**. Insurance companies are governed by the Solvency II directive, which mandates a risk-based capital framework requiring a Minimum Capital Requirement (MCR) and a higher Solvency Capital Requirement (SCR).

Pension Funds in Germany are subject to the IORP II Directive, focusing on governance, information to members, and prudential requirements, but generally with less onerous solvency requirements than Solvency II.

Hedge Funds in USA. Hedge fund managers are largely regulated by the SEC under the Investment Advisers Act. While the funds themselves generally don't have minimum capital requirements (they are for "accredited investors"), the managers may need minimum capital and are subject to extensive registration, reporting, disclosure, and compliance requirements.

Insurance in Canada. The regulatory framework is the Minimum Capital Test (MCT) for P&C insurers and the Life Insurance Capital Adequacy Test (LICAT) for life insurers, which sets a minimum ratio (e.g., 100% for MCT) and a supervisory target (e.g., 150% for MCT).

Below is the estimated regulatory capital needed for each entity of PFH.

1. **Perun Bank (PEB).**

Capital Ratio Requirement (CRR) under Basel III/3.1, for example 14%-16% of Risk-Weighted Assets (RWA), is estimated between 15 000 000 000 USD and 50 000 000 000 USD.

2. **Perun Assurance (PEA).**

Solvency Capital Requirement (SCR) under Solvency II (2026), typically 100-200% of Minimum Capital Requirement (MCR) is estimated to be between 5 000 000 000 USD and 20 000 000 000 USD. It is highly dependent on the volume and risk profile of the insurance policies underwritten (e.g., catastrophe exposure).

3. **Perun Pensions (PEP).**

Operational Capital under IORP II (EU) is generally lower as the fund manages assets (AUM), not balance sheet risk, but reserves are required.They are estimated between 500 000 0000 USD and 1000 000 000 USD. The Operational Capital covers operational risk, guarantees, and regulatory thresholds.

4. **Perun Capital (PEC).**

AIFMD/IFR (Investment Firm Regulation). Capital is based on fixed overheads and Assets Under Management (AUM) is estimated between 50 000 000 USD and 200 000 000 USD. Covers fixed overheads (e.g. 25% of FOH) and specific K-factors for investment firms.

5. **Perun Finance Holding (PFH).**

Consolidated Capital Buffer (Holding Company Liquidity and Buffer) is estimated between 1 000 000 000 and 5 000 000 000 USD. These are funds for emergency capital support, strategic investments, and M&A.

A successful global financial group with universal banking, major insurance, and large asset management arms would require an initial, well-cushioned capital base in the range of 22 000 000 000 USD to 76 000 000 000 to meet regulatory requirements, absorb unexpected losses, and establish a credible position in global markets.

This estimate is a broad range for regulatory capital (Tier 1/Own Funds) and operational reserves to run successfully across the stipulated jurisdictions.

General strategy for creating Perun Financial Holding (PFH)

Here we provide a general strategy for creating international financial group PFH containing a bank, a pension fund, an insurer, a financial brocker and an insurance brocker based in Sofia, Bulgaria, with presence in China, USA, Germany, UK, Rusiia, India, Indonesia, Canada, Australia, Brazil, Argentina, Spain, Switzerland, Turkey, Republic of South Africa (RSA), in total, 16 countries.

The name of the company is Perun Finance Holding (PFH). The headquarters is in Sofia, Bulgaria. The legal structure is Financial Holding Company. Target Launch: 2026-2036. Core Business Units: Commercial & Investment Bank, Pension Fund Management, Insurance Company, Financial Brokerage, Insurance Brokerage. Geographic Presence: 16 countries across 6 continents

Budget structure for creating Perun Finance Holding (PFH).

Total Capital Required is between 2.8 and 3.5 billion euro. It is distributed in time and on organizational structure as follows.

Phase 1: Establishment (years 1-5)

1. Corporate Structure & Legal Formation

Holding Company Formation (Bulgaria). Needed 0.5 - 1.0 million euro for registration, legal structure, corporate governance.

Subsidiary Registrations (16 countries). Needed 3.0 - 5.0 million euro for legal entity formation in each jurisdiction.

Legal & Consulting Fees. Needed 8.0 - 12.0 million euro for international law firms, regulatory consultants.

Licensing Applications. Needed 15.0 - 25.0 million euro for banking, insurance, brokerage licenses.

Compliance Infrastructure. Needed 5.0 - 8.0 million euro for AML/KYC systems, regulatory reporting.

Corporate Governance Setup. Needed 2.0 - 3.0 million euro for board structure, policies, procedures.

Intellectual Property. Needed 1.5 - 2.5 milllion euro for trademarks, branding, technology patents.

Subtotal for Phase 1: 35 - 56.5 million euro.

2. Regulatory Capital Requirements by Entity.

A. Commercial & Investment Bank

Bulgaria (HQ). Minimum capital needed is 50 - 100 million euro. Regulatory body is Bulgarian National Bank (BNB).

Germany. Minimum capital needed is 100 – 150 million euro. Regulatory body is BaFin.

UK. Minimum capital needed is 80 – 120 million euro. Regulatory bodies are FCA & PRA.

USA. Minimum capital needed is 150 – 250 million euro. Regulatory bodies are OCC, Federal Reserve, FDIC.

Switzerland. Minimum capital needed is 100 – 150 milllion euro.Regulatory body is FINMA.

China. Minimum capital needed is 200–300 millions euro. Regulatory body is CBIRC (requires local partnership).

Russia. Minimum capital needed is 50–80 millions euro. Regulatory body is Central Bank of Russia.

India. Minimum capital needed is 60–100 millions euro. Regulatory body is RBI.

Indonesia. Minimum capital needed is 40–60 millions euro. Regulatory body is OJK.

Canada. Minimum capital needed is 80-120 millions euro. Regulatory body isOSFI.

Australia. Minimum capital needed is 70-100 millions euro. Regulatory body is APRA.

Brazil. Minimum capital needed is 50 - 80 millions euro. Regulatory body is Banco Central do Brasil.

Argentina. Minimum capital needed is 30-50 millions euro. Regulatory body is BCRA.

Spain. Minimum capital needed is 80-120 millions euro. Regulatory body is Banco de España.

Turkey. Minimum capital needed is 40-60 millions euro. Regulatory body is BDDK.

South Africa. Minimum capital needed is 40-60 millions euro. Regulatory body is SARB.

Banking Capital Subtotal: 1,220 - 1,900 Million euro.

B. Insurance Company

Bulgaria (HQ). Minimum capital needed 15 – 25 million euro. The regulatory body is Financial Supervision Commission.

Germany. Minimum capital needed 30 – 50 million euro. The regulatory body is BaFin.

UK. Minimum capital needed is 25 – 40 milllion euro. The regulatory bodies are FCA & PRA.
USA. Minimum capital needed 50 – 80 million euro. The regulatory body is State Insurance Departments.
Switzerland. Minimum capital needed 30 – 50 million euro.The regulatory body is FINMA.
Other Markets. Minimum capital (avg) 15 – 30 million euro each. Various regulators.
Insurance Capital Subtotal: 350 - 550 million euro.
C. Pension Fund Management
Bulgaria (HQ). Minimum capital needed 5 – 10 milllion euro. Regulatory body FSC regulation.
Major Markets. Minimum capital needed 80 – 150 million euro. EU, USA, UK, Switzerland.
Emerging Markets. Minimum capital needed 40 – 70 million euro. Other jurisdictions
Pension Fund Capital Subtotal: 125 - 230 million euro.
D. Financial Brokerage
Securities Licenses. Cost 50 – 80 million euro. All 16 jurisdictions.
Trading Infrastructure. Cost 30 – 50 million euro. Technology, connectivity.
Regulatory Capital. Cost 80 – 120 million euro. Regulatory body varies by jurisdiction.
Financial Brokerage Subtotal: 160 - 250 million euro.
E. Insurance Brokerage
Licenses & Registrations. Cost 15 – 25 million euro. All jurisdictions.
Professional Indemnity. Cost 5 – 10 million euro. Insurance coverage.
Operating Capital. Cost 20 – 35 million euro. Initial operations.
Insurance Brokerage Subtotal: 40 - 70 million euro.
3. Technology & Infrastructure
Core Banking System. Cost 80 – 120 million euro. Temenos, FIS, Oracle Flexcube.
Insurance Platform. Cost 30 – 50 million euro. Guidewire, Duck Creek.
Trading Platform. Cost 40 – 60 million euro. Bloomberg, Reuters integration.
Pension Management System. Cost 20 – 35 million euro. Specialized pension software.
Cybersecurity Infrastructure. Cost 25 – 40 million euro. SOC, firewalls, encryption.
Data Centers. Cost 50 – 80 million euro. Primary + DR sites
Cloud Infrastructure. Cost 15 – 25 million euro. AWS, Azure, hybrid model.
Mobile & Web Applications. Cost 20 – 35 million euro. Customer-facing platforms.
AI & Analytics Platform. Cost 15 – 25 million euro. Risk management, fraud detection.
Integration & APIs. Cost 10 – 20 million euro. System connectivity.
Disaster Recovery. Cost 15 - 25million euro. Backup systems.
Technology Subtotal: 320 - 515 million euro.
4. Physical Infrastructure
Sofia (HQ). Purchase/Build 15,000 sqm. Cost 25 – 40 million euro.
Frankfurt. Office 3,000 sqm. Cost 8 – 12 million euro.
London. Office 3,000 sqm . Cost 12 – 18 million euro.
New York. Office 2,500 sqm. Cost 15 – 25 million euro.
Shanghai. Office 2,500 sqm. Cost 10 – 15 million euro.
Zurich. Office 2,000 sqm. Cost 10 – 15 million euro.
Other Locations (10). Offices 1,000-2,000 sqm each. Cost 40 – 70 milllion euro.
Renovation & Fit-out. All locations. Cost 30 – 50 million euro.
Furniture & Equipment. All locations. Cost 15 – 25 million euro.
Physical Infrastructure Subtotal: 165 - 270 million euro.
5. Human Resources - Initial Staffing
Executive Management. 25 employees. Cost 8 – 12 million euro per year, starting from 4th year.
Banking Operations. 450 employees. Cost 35 – 50 million euro per year, starting from 4th year.
Insurance Operations. 200 employees. Cost 15 – 22 milllion euro per year, starting from 4th year.
Brokerage Operations. 150 employees. Cost 12 – 18 milllion per year, starting from 4th year.
Technology & IT. 180 employees. Cost 18 – 28 million euro per year, starting from 4th year.
Compliance & Legal. 120 employees. Cost 15 – 22 milllion euro per year, starting from 4th year.
Risk Management. 80 employees. Cost 10 – 15 million euro per year, starting from 4th year.
Finance & Accounting. 100 employees. Cost 10 – 15 million per year, starting from 4th year.
HR & Administration. 70 employees. Cost 5 – 8 million per year, starting from 4th year.
Marketing & Sales. 200employees. Cost 15 – 22 million euro per year, starting from 4th year.
Customer Service. 250 employees. Cost 12 – 18 million euro per year, starting from 4th year.
Total Initial Staff: approximately 1 825.
HR Subtotal (Years 1-5): 310 - 460 million euro.
6. Marketing & Brand Launch

Brand Development. Cost 5 – 8 milllion euro. Logo, identity, guidelines.
Website & Digital Presence. Cost 3 – 5 million euro. Multi-language, multi-country.
Launch Campaign (Global). Cost 40 – 60 million euro. TV, digital, print, events.
Regional Marketing. Cost 25 – 40 million euro. Localized campaigns.
Sponsorships & Partnerships. Cost 15 – 25 million euro. Sports, cultural events.
PR & Communications. Cost 8 – 12 million euro. Media relations, crisis management.
Customer Acquisition. Cost 30 – 50 million euro. Incentives, promotions.
Marketing Subtotal: 126 - 200 million euro.
7. Operating Expenses (Years 1-5)
Office Operations. Cost 20 – 30 million euro. Utilities, maintenance, supplies
Professional Services. Cost 25 – 40 million euro. Auditors, consultants, advisors.
Insurance & Bonding. Cost 15 – 25 milllion euro. D&O, E&O, cyber insurance.
Travel & Logistics. Cost 10 – 15 milllion euro. International coordination.
Training & Development. Cost 12 – 18 million euro. Staff training, certifications.
Contingency Reserve. Cost 50 – 80 milllion euro. Unexpected costs
Operating Expenses Subtotal: 132 - 208 mMillion
Phase 1 total: 2,768 - 4,459 million euro.
Recommended Initial Capital Raise: 3.0 - 3.5 billion euro.

Phase 2: Growth and expansion (years 6-10).

Additional Capital Requirements
Market Expansion. Cost 200 – 350 milllion euro. New products, services.
Technology Upgrades. Cost 100 – 150 milllion euro. System enhancements
Marketing & Growth. Cost 150 – 250 million euro. Customer acquisition.
Additional Staff. 250 – 400 employees. Growth to 3,500+ employees.
Acquisitions. Cost 300 – 500 million euro. Strategic purchases
Capital Buffer. Cost 200 – 350 million euro. Regulatory requirements
Phase 2 Total: 1,200 - 2,000 million euro.

Funding possibilities for creating Perun Finance Holding (PFH)

Option 1: Private Equity Consortium (Recommended)

Structure: 3.0 - 3.5 billion euro Initial Capital. Here, there are several possibilities for the investors:

1. Lead PE Fund which could ensure 30-35% share or 900 - 1,225 million euro. Examples for PE Fund are: Blackstone, KKR, Carlyle.

2. Sovereign Wealth Funds which could ensure 20-25% share or 600 – 875 million euro. Examples for Sovereign Wealth Funds are: GIC, ADIA, CIC, RDIF.

3. Strategic Investors which could ensure 15-20% shere or 450 – 700 million euro. Examples for strategic investors are existing financial institutions.

4. Family Offices which could ensure 10-15% share or 300 – 525 million euro. Examples are ultra-high net worth families.

5. Founders/Management which could ensure 15-20% share or 450 – 700 milllion euro. Examples are Sweat equity + investment.

Advantages of this option are:
1. Experienced financial investors
2. Strategic guidance and networks
3. Flexible terms
4. Long-term perspective (7-10 years)
Timeline: 12-18 months fundraising

Option 2: Strategic Partnership Model

The founders should become partners with existing financial institutions. Potential partners should be:

1. Major Bank like UniCredit, ING, Standard Chartered with expertise in banking operations which could ensure 25-30% stake.

2. Insurance Group like Allianz, AXA, Zurich with expertise in insurance operations which can ensure 20-25% stake.

3. Asset Manager Pension/investments like BlackRock, Vanguard, Fidelity which could ensure 15-20% stake.

4. Technology Partner like FIS, Temenos, Oracle with expertise in IT infrastructure which could ensure 10-15% stake.

5. Regional Partners which could bring local expertise and could ensure 20-25% stake.

Advantages of this option are:
1. Operational expertise.
2. Existing infrastructure.

3. Customer base access.
4. Regulatory experience.

Option 3: Hybrid Public-Private Model

Structure of financing with this option could be:

1. Private Placement: 2.0 - 2.5 billion euro (60-70%).
2. Government Support: 500 - 700 million euro (15-20%).
3. IPO (Year 3-4): 500 - 800 million euro (15-20%).

The options for Government Support could be:

1. From Bulgarian Government - strategic investment, tax incentives. This could bring 100 – 200 milllion euro.
2. EU Funds – from Development programs . This could bring 150 – 250 million euro.
3. Export Credit Agencies - guarantees, loans. This could bring 150 – 200 milllion euro.
4. Development Banks - EIB, EBRD, IFC loans. This could bring 100 – 150 milllion euro.

Advantages of this option are:

1. Lower cost of capital.
2. Government support and goodwill.
3. Public market access for liquidity.
4. Enhanced credibility.

Option 4: Phased Regional Rollout.

The PFH establishnment should start with core markets, and expand gradually. There should be several phases:

1. Phase 1 (Years 1-4): investing 800 million - 1.2 billion euro. The countries where the establishment should start are: Bulgaria (HQ), Germany/France, UK, USA (limited).
2. Phase 2 (Years 5-7): 1.0 - 1.5 billion euro. The countries in this phase are: Switzerland, Spain, Canada, Australia.
3. Phase 3 (Years 8-10): 800 million - 1.2 billion euro. The countries here should be: China, India, Russia,Brazil, Other markets.

Advantages of this option are:

1. Lower initial capital requirement.
2. Learn and adapt.
3. Prove concept before expansion.
4. Easier to raise follow-on capital.

Recommended funding structure

Optimal Approach for funding is the Hybrid Strategy to obtain Initial Capital of 3.2 billion euro as follows:

1. Private Equity Lead: 1.0 billion euro (31%).
2. Sovereign Wealth Funds: 800 million euro (25%).
3. Strategic Financial Partners: 600 million euro (19%).
4. Family Offices/UHNW: 400 million euro (12.5%).
5. Founders/Management: 400 million euro (12.5%).

Additional Funding (Years 6-10):

Retained Earnings: 200 - 400 million euro.
Debt Financing: 300 - 500 million euro.
Secondary Offering/IPO: 500 - 800 million euro.

Revenue projection

Year 1-5 (Establishment)

Revenue: 150 - 250 million euro.
EBITDA: -200 to -100 million euro (investment phase).

Year 6-9

Revenue: 800 million - 1.2 billion euro.
EBITDA: 50 - 100 million euro.

Year 10 Year 10 (Mature State)

Revenue: 8 - 12 billion euro.
EBITDA: 2.0 - 3.0 billion euro.
Net Income: 1.2 - 2.0 billion euro.
ROE: 15-20%

Business units revenue breakdown (Year 9) should be:

Banking : 1,200 - 1,800 million euro - 40-50% of total.
Insurance: 700 - 1,000 million euro - 25-30% of total.
Pension Funds: 300 – 500 million euro - 10-15% of total.

Financial Brokerage: 250 – 400 milllion euro - 8-12% of total.

Insurance Brokerage: 150 – 250 million of euro - 5-8% of total.

Risk analysis and mitigation.

The Major Risks with their impact, probability and possible mitigation, are:

1. Regulatory Rejection. Impact - critical. Probability – medium. Mitigation measures – working with experienced advisors, strong compliance.

2. Capital Shortfall. Impact – High. Probability – Medium. Mitigation measures – applying phased approach, multiple funding sources.

3. Market Competition. Impact – High. Probability - High. Mitigation - differentiation, technology advantage.

4. Economic Downturn. Impact –High. Probability – Medium. Mitigation - diversified geography, conservative reserves.

5. Cybersecurity Breach. Impact - Critical.Probability-Medium. Mitigation - Best-in-class security, insurance.

6. Geopolitical Risk. Impact – Medium. Probability - Medium. Mitigation - Diversification, local partnerships.

7. Talent Acquisition. Impact –Medium. Probability – High. Mitigation - Competitive compensation, culture.

8. Technology Failure. Impact – High. Probability-Low. Mitigation - Redundancy, proven systems

Market entry strategy by country

Tier 1 Priority (Years 1-5)

1. Bulgaria (HQ). Rationale: EU member, lower costs, strategic location. Initial Investment: 150 - 250 million euro. Strategy: Full service launch, flagship market.

2. Germany/France. Rationale: Largest EU economy, financial hub. Initial Investment: 200 - 350 million euro. Strategy: Focus on corporate banking, insurance

3. UK. Rationale: Global financial center, English-speaking. Initial Investment: 180 - 300 million euro. Strategy: Investment banking, wealth management.

4. USA. Rationale: Largest market, highest potential. Initial Investment: 250 - 400 million euro. Strategy: Start with brokerage, expand to banking

Tier 2 Priority (Years 5-7). Switzerland, Spain, Canada, Australia. Combined Investment: 400 - 700 million euro. Strategy: Selective services, partnerships.

Tier 3 Priority (Years 8-10). China, Russia, India, Indonesia, Brazil, Argentina, Turkey, South Africa. Combined Investment: 500 - 900 million euro. Strategy: Joint ventures, local partnerships, gradual expansion.

Regulatory compliance roadmap.

In EU (incl. Bulgaria, Germany, France, Spain), primary regulations are: CRD VI, MiFID II, Solvency II, GDPR. Timeline is 12-18 months. Complexity is High.

In UK, primary regulations are: PRA/FCA rules, post-Brexit regime. Timeline is 12-18 months. Complexity is High.

In USA, primary regulations are: Dodd-Frank, Bank Holding Company Act. Timeline is 18-24 months. Complexity is Very High.

In Switzerland, primary regulations are: Banking Act, FINMA regulations. Timeline is 12-18 months. Complexity is High.

In China, primary regulations are: Foreign ownership restrictions. Timeline is 24-36 months. Complexity is Very High.

In Russia, primary regulations are: Central Bank regulations. Timeline is 12-18 months. Complexity is Medium-High.

In Others, primary regulation is: Local banking/insurance laws. Timeline is 12-24 months. Complexity is Medium.

Governance structure.

The governance structure of PFH should include:

1. Board of Directors (15 members)
2. Executive Directors: 2 (CEO + CFO)
3. Chairman: Independent, financial services experience
4. Independent Directors: 8 (majority)
5. Investor Representatives: 4

The positions, salary and main responsibilities of executive management should be:

1. Group CEO. 800K - 1.5M euro.Overall strategy and operations.
2. CFO. 600K - 1.0M euro. Finance, treasury, investor relations.
3. CRO. 500K - 900K euro. Risk management across all entities.
4. COO. 500K - 900K euro. Operations, technology, processes.

5. Chief Compliance Officer. 400K - 700K euro. Regulatory compliance, AML.
6. Banking CEO. 500K - 900K euro. Banking operations.
7. Insurance CEO. 400K - 700K euro. Insurance operations.
8. CTO. 400K - 700K euro. Technology and innovation.
9. General Counsel. 400K - 700K euro. Legal affairs.
10. Chief HR Officer. 300K - 500K euro. Human resources.

Detailed implementation timeline

Implementation should pass through several phases:

1. Pre-Launch Phase (Months 1-12)
    1.1. Secure founding capital (500 million – 1billion euro initial tranche)
    1.2. Establish holding company in Bulgaria
    1.3. Hire core executive team
    1.4. Engage regulatory consultants
    1.5. Begin license applications
    1.6. Develop business plan details
2. Foundation Phase (Months 13-60)
    2.1. Complete capital raising (3.0 - 3.5 billion euro)
    2.2. Obtain key licenses (Bulgaria, Germany, UK)
    2.3. Build technology infrastructure
    2.4. Hire management teams (500+ staff)
    2.5. Establish partnerships
    2.6. Develop products and services
3. Launch Phase (Months 60-84)
    3.1.Soft launch in Bulgaria
    3.2. Launch in Germany/France and UK
    3.3.Begin USA operations (brokerage)
    3.4. Marketing campaign
    3.5. Customer acquisition
    3.6. Achieve first revenues
4. Growth Phase (Years 8-10)
    4.1. Expand to Tier 2 markets
    4.2. Achieve profitability (Year 8)
    4.3. Launch additional products
    4.4. Consider IPO (Year 4, Year 10)
    4.5. Begin Tier 3 market entry
    4.6. Scale to 3,500+ employees

Success factors and KPIs

Critical Success Factors

Regulatory Approval: Securing all necessary licenses
Capital Adequacy: Maintaining strong capital ratios
Technology Excellence: Best-in-class digital platforms
Talent Acquisition: Attracting top financial professionals
Brand Building: Establishing credibility and trust
Risk Management: Robust controls and governance
Customer Focus: Excellent service and products
Partnerships: Strategic alliances in key markets
Culture: Proactive regulatory compliance
Innovation: Differentiation through technology and service

Key performance indicators (KPIs)

Financial KPIs
Return on Equity (ROE): Target 15-20% by Year 5
Cost-to-Income Ratio: Target <60% by Year 5
Net Interest Margin: Target 2.5-3.5%
Capital Adequacy Ratio: Maintain >12% (Basel III)
Solvency Ratio (Insurance): Maintain >150%
Operational KPIs
Customer Acquisition: 500K+ by Year 5

Digital Adoption: >70% of transactions
Employee Satisfaction: >80%
NPS (Net Promoter Score): >50
Assets Under Management: 20-30 billion euro by Year 10.

Conclusions and recommendations

Feasibility Assessment: challenging but achievable.
Strengths: Diversified business model, Global presence from inception, Bulgaria as cost-effective EU base, multi-channel revenue streams
Challenges: Massive capital requirement, Complex regulatory environment, Intense competition in all markets, Long path to profitability
Recommended Approach:
1. Start with Phased Rollout: Focus on 4-5 core markets initially
2. Secure Anchor Investors: Major PE fund + sovereign wealth fund
3. Strategic Partnerships: Partner with established institutions
4. Technology First: Invest heavily in digital capabilities
5. Regulatory Excellence: Hire best compliance talent
6. Patient Capital: Ensure investors have 7-10 year horizon
Alternative Strategy:
Regional Focus. If 3 and more billion euro is challenging to raise, consider focusing on on CEE/Balkans initially: Bulgaria, Macedonia, Romania, Serbia, Greece.
Required Capital: 500-800 million euro
Expand gradually: Add Western Europe, then global
Timeline: 3-5 years to profitability
Next steps:
Engage Investment Bank: For capital raising advisory
Hire Regulatory Consultants: For license applications
Develop Detailed Business Plan: 100+ page document
Create Financial Model: 10-year projections
Begin Investor Outreach: Target PE funds and SWFs
Assemble Founding Team: CEO, CFO, CRO minimum
Conduct Market Research: Validate assumptions
Establish Legal Structure: Holding company formation
This is an ambitious but potentially transformative project. Success requires exceptional execution, patient capital, and world-class talent. The financial services industry needs innovative, technology-forward players, and this structure could fill that gap.

References

APRA (2026) Australia's Prudential Authority https://www.apra.gov.au
Basel III (2026) https://www.bis.org/bcbs/basel3.htm
CBIRC (2026) China Banking and Insurance Regulatory Commission - Financial Regulatory Administration of China https://www.nfra.gov.cb/en
Banco Central do Brasil (2026) https://www.bcb.gov.br/en
CBR (2026) Central Bank of Russia https://www.cbr.ru/eng
ECB (2026) European Central Bank www.ecb.europa.eu
FCA(2026) Financial Conduct Authority www.fca,org,uk
FDIC (2026) Federal Deposit Insurance Cooperation https://www.fdic.gov
FED (2026) Federal Reserve of USA https://www.federalreserve.gov
FINMA (2026) Swiss Financial Market Supervisory Authority https://www.finma.ch/en
OCC (2026) Office of the Comptroller of the Currency of USA https://www.occ.treas.gov
OJK (2026) Otoritas Jasa Keuangan Financial Service Authority of Indonesia https://ojk.go.id/en/default.aspx
OSFI (2026) Office of the Superintendant of Financial Institutions of Canada https://www.osfi-bsif.gc.ca/en
PRA(2026) Prudential Regulation Authority www.bankofengland.co.uk/prudential-regulation
RBI (2026) Reserve Bank of India https://www.rbi.org.in

Solvency II (2026) https://www.eiopa.europa.eu/browse/regulation-and-policy/solvency-ii_en

Zermelo-Fraenkel set theory (ZF) – a study on the essential axioms for a better application of the foundational theory

Yousef Methkal Abd Algani,
The Academic Arab College for Education in Israel, Israel
Yosefabdalgani@gmail.com

Abstract.

Set theories, such as Zermelo-Fraenkel's, are frequently represented as a synthesis of 2 types of principles: set-theoretic and logical principles. Although its importance as a mathematical basis, it has two drawbacks: it can't be put down directly because of its infinite number of axioms, and it has a countable model because of the Löwenheim–Skolem theorem. This study illustrated how minimal tinkering with analytical theorems and definitions was essential to carried it out in a constructive manner. Furthermore, it is shown that, unlike IZF (Intuitionistic Zermelo-Fraenkel), the strength of ZF improves when an axiom declaring that the trichotomous ordinals form a set is added. This study highlight three key elements of this formalized inference framework: (i) the underlying logical equations is designed to provide strategic directions and reasoning with gaps. (ii) The mathematical framework is a hierarchically organised system of conceptions and processes, and it is a definitional improvement of Zermelo-Fraenkel set theory. (iii) Through axioms for lemmas and descriptions the development of formal proofs is inextricably linked to the framework. We show that Zermelo-Fraenkel set theory has an axiomatized property with regard to any intermediate logic defined by a set of axiom frameworks.

Keywords: set theory, Zermelo-Fraenkel, axioms, category-theory, foundations of math.

Introduction

The Zermelo-Fraenkel set theory is the most commonly recognised mathematical basic theory (ZF). Although there is a full explanation in the research, it's worth remembering that ZF has (7) axioms and 2 infinite axiom strategies: the (7) axioms such as the Axiom of Regularity (REG), the Sumset Axiom (SUM), the Infinite Set Axiom (INF), Extensionality Axiom (EXT), the Pair Axiom (PAIR), and the Empty Set Axiom (EMPTY); and the 2 axioms arrangements are the Substitution Axiom Scheme (SUB) and the Separation Axiom Scheme (SEP). Overall, ZF is an endlessly axiomatized theory; also, a very well Montague theorem argues that a limited axiomatization of ZF in the language of ZF is not conceivable without admitting additional objects (Bendkowski 2020). Furthermore, these 2 models either break from Zermelo-Fraenkel's ontology or employ non-constructive axioms1: NGB employs classes, but CVN begins with the postulate that there is a set that encompasses all sets (Zakharov and Rodionov 2018). Alternate models would not be considered; the goal of this note is to demonstrate that, although keeping ZF's axiom 'everything is a set', a limited axiomatization of set theory is conceivable using a unique creative axiom that employs a new idea of global quantification.

The reality that Zermelo-Fraenkel set theory (ZF) could be used as a groundwork for practically all of advanced mathematics is a mathematical reality and this study should begin by emphasising that this reality will be taken unquestioned throughout the rest of the paragraph – there is no mathematical inspiration to substitute ZF (Fernengel and Koepke 2016). Moreover, the Elementary Process Theory (EPT) has subsequently been established as a formalized axiomatic approach which could be used as a fundamental structure for physicists if antimatter and matter repel one another gravitationally (Cabbolet and de Swart 2014a; Wehr 2020, Passmann 2019a; Abd Algani, 2025). The research on the Elementary Process Theory entailed looking for fundamental concepts that are coherent with the major consequences of the postulated presence of gravitational attraction (Makarov 2021, FREIRE, n.d.), which is that antimatter must have negative gravitational mass and positive rest mass.

Every attempts to formalise the EPT inside the context of Zermelo-Fraenkel, that is, in the regular language of mathematics, is now likely to be met with opposition (Crosilla 2019; Shin 2019). The property of ZF, that anything must be a set, leads to philosophical, not mathematical, problems that are both intractable and unavoidable within the context of ZF2. There are only two alternatives left to pick from:

- One can keep Zermelo-Fraenkel, which means accepting that the formalisation of the Elementary Process Theory is fraught with issues.
- The concerns with the formalisation of the EPT can be resolved, implying that ZF must be rejected on philosophic grounds.

ZFP adds ∈ with 2 additional binary predicated symbols, π1 and π2, which indicate "is first projection of" as well as "is second projection of," respectively. Set(x) and Pair(x) are shorthand for formulations that differentiate ordered pairs and sets by the principle that an ordered pair has a first projected while a set does not. The axioms of ZFP are divided into 2 categories: ordered pairs and sets (Boulabiar 2015, Freiman 2020). By changing the axioms of ZF and constraining quantifiers utilizing Set(x) in the correct locations, we were managed to produce almost all of ZFP's axioms for sets. To accommodate ordered pairs and sets at the same time, the Foundational axiom had to be changed (Herzberg 2016, Assaf et al. 2016). The anticipated abstract qualities of ordered pairs are specified by ZFP's axioms, which include the fact that ordered pairs have no sets elements (Garcia Tarrach 2017, Dunne, Wells, and Kamareddine 2020a). To show that ZFP is coherent if ZF is, we build a system in ZF and show that it fulfils ZFP's axioms. Since there can be additional non-set items at each level of the construction, developing a models for a set theory with non-set objects with 'internal' architecture which can involve sets varies from developing a prototype for a set theories with no urelements or just basic urelements (IMAMURA, n.d.). Our model's domain W is analogous to the Von Neumann hierarchy's domain V(Erkan 2020). The ability set of the preceding tiers is used to build each tier of V. When creating the tiers of W, the disjoint total of the Cartesian product (Wα) 2 and the power set P (Wα) and is used to create every succession tier (Passmann 2019b).

The residual of this document is organized as follows; Section 2 defines the related works on axiomatic set theory of fundamental theory of mathematics using various axioms. Section 3 tells about the Theoretical set of Axioms and Standard functional-Theory Axioms of the proposed model. Section 4 shows the discussion of the theorem works in detail. Section 5 finally concludes the study.

Related works

In (Cabbolet 2021; Abd Algani **&** Amal Sharif- Rasslan, 2023), explained about the axiomatic set theory and its non-classic first order theory, Although its importance as a fundamental concept for mathematics, Zermelo-Fraenkel Set Theory (ZF) has 2 major drawbacks: it can't be written down directly because of it's endlessly several more axioms, but it has a quantifiable structure according to the Löwenheim–Skolem theorem. This study explains the axioms that must be accepted in order to eliminate these 2 characteristics. Moreover, the axioms of categorical theories are demonstrated to be true, implying that the current world can be used as a philosophical foundation for category theory. Furthermore, none of the completeness and soundness properties have been tested for the current theory, leading to the unavoidable final result that only more study will be capable to determine if the current findings represent a breakthrough in mathematical fundamentals.

In (Christopoulou 2020) describes about the abstraction principles of axioms system defining mathematical terms. This section argues that mathematical latent assumptions, such as axiom sets and simplification rules, support several features of mathematics diversity. The study, on the other hand, shifts the emphasis of attention to an architectural perspective on mathematics latent assumptions. An architectural analysis of mathematics latent concepts backs up a modest pluralist viewpoint. Fundamentals are uninteresting to a structuralist. The study considers the situation of satisfied abstraction rules in the final part. These rules are assumed to determine specific architectures implicitly. Specifically, satisfied abstraction rules that contradict one another, such as the Nuisance Principle and Hume's Principle, define alternate topologies with various example object networks.

In (Dunne, Wells, and Kamareddine 2020b; Abd Algani, 2021; 2024) define that add an abstract obstacle to Zermelo-Fraenkel ST. Set theory, generally Zermelo-Fraenkel set theory (ZF) or one of its variations, underpins a lot of mathematical literature, whether directly or intuitively. Each mathematical entity in Zermelo-Fraenkel must be a set because the domain of discourse only includes sets. These "accidental theorems" don't correspond to how people understand regarding mathematical, and they also present problems when employing set theory in machine-assisted theorem demonstrating. Without any concept of representations, ZFP has a better organic and abstract axiomatic description of ordered pairs. This work gives ZFP axioms as well as an evidence of the availability of a prototype for ZFP in Zermelo-Fraenkel, which indicates that ZFP is coherent if ZF. The method utilised to introduce this abstraction obstacle to Zermelo-Fraenkel (ZF) is discussed.

In (Paulson 2019) explains about the higher order logic ZF set theory. This item is a fresh Isabelle/HOL formalisation of Zermelo-FraenkelC set theory. It's conceptually equal to Obua's HOLZF, with the goal of achieving the tightest feasible unification. Sets are classified as type V, and the method elts :: V ⇒ V set maps a set to its components. Kirby provides a description of multiplying and adding for generic collections (not just ordinals). The research contains important cardinal mathematical conclusions. There are a lot of conclusions

when it comes to order kinds. With the goal of fostering improvements that incorporate V with other Isabelle/HOL types, the hypothesis offers 2 types classes: ebeddable, which is a class of types that can be infused into V (along with V on its own and V list, V*V, and so on), and small, which is a class of types that corresponds to several Zermelo-Fraenkel set.

In (Jia 2017; Abd Algani, 2021) describes that the various axioms of choices in ZF topologies. Cantor thought that characteristics that apply to limited sets could apply to limitless sets as well. This apparently basic concept has sparked debate since it has magical properties in mathematical in generally and topological in particularly. In this study, we'll go over some basic topological ideas like continuation; we would prove that the Axiom of Choice is equal to Tychonoff's Theorem through demonstrations. We'll check over Kelley's first evidence and make any necessary corrections. They additionally present the Boolean Prime Ideal Theorem (a lesser form of the Axiom of Choice) for Hausdorff spaces, which is identical to Tychonoff's Theorem. Lastly, we'll examine at one of the Axiom of Choice's topology repercussions: Stone-Cech Compactification. The Stone-Cech Compactification is made up of ultra-filters, the existence of which is determined by the Axiom of Choice.

Theoretical set of Axioms

Firstly and foremost, a statement. The logical characters $'\forall'$ and $'\exists'$ as well as configurations such as$'\forall x'$, $'\exists y'$containing of such a character plus variables are referred to as 'quantifiers' in ordinary first-order logic. Although this might not be an issue in conventional reasoning, the logical characters $'\forall'$ and $'\exists'$ are used in several constants below. To prevent misunderstanding (Sieg and Walsh 2021) (Bingham and Ostaszewski 2019), we shall refer to the character $'\exists'$ as an 'existential quantification symbol' and the sign $'\forall'$ as a 'universal quantification symbol' (Antos et al., n.d.; Abd Algani, 2023).

Axiom 1. The Extensionality Axiom for Sets (EXT) states that 2 sets X and Y are similar if their components were identical (functions and sets).

$$\forall X \forall Y \, (X = Y \Leftrightarrow \forall \alpha \, (\alpha \in X \Leftrightarrow \alpha \in Y)) \tag{1}$$

Axiom 2. The Pairing Axiom (PAIR) states that for each (a) and each (b), there exists a set X that has exactly the a and b as its constituents.

$$\forall \alpha \forall \beta \exists X \forall \gamma (\gamma \in X \Leftrightarrow \gamma = \alpha \, V \, \gamma = \beta) \tag{2}$$

Remark 1. The empty set could be understood as the individuality {} in the planned framework utilizing set-builder notation. Additionally, assuming particular items $\underline{\alpha}$ and $\underline{\beta}$ in the desired framework's universe, the pair set of $\underline{\alpha}$ and $\underline{\beta}$ can be recognized with the individual$\{\alpha = \beta\}$. If $\underline{\alpha}$ = , then this is a singleton (Cavitt 2017). □

Remark 2. The ensembles of this theory can't be regarded as multi-sets, objects in multi-set theory. As a part of this א theory, we have for everything one in the category of functions and sets which$\{\propto\} = \{\propto, \propto, \propto\}$. That's not the situation with multi-set theory, where a component can appear multiple times in a multi-set. Then there's $\{\propto, \propto\} \neq \{\propto, \propto, \propto\}$ for multi-sets $\{\propto, \propto\}$ and$\{\propto, \propto, \propto\}$.

Definition 1: When *t* is a term, therefore $(t)^+$ is a term, which is referred to as 'the singleton of *t*'—which can be represented as $(t)^+$ if there is no uncertainty. If $\propto$ is a variable that spans all things and *x* is a variable that spans all sets, then $\propto^+$ is a variable that spans all singletons and x^+ is a variable that spans all single people of sets (Crosilla 2019). As a result,

$$\forall \alpha \forall \beta (\beta = \alpha^+ \Leftrightarrow \exists X (\beta = X \wedge \forall \gamma (\gamma \in X \Leftrightarrow \gamma = \alpha))) \tag{3}$$

Similarly forx^+,

Notation 1. The parameters$\emptyset^+, \emptyset^{++}, \emptyset^{+++}$,... are included in the language $L_{\aleph}$ because of Definition 1. As a result, we could use the (limited) Zermelo-Fraenkel ordinals as a notation for these singletons at this instant:

$$\begin{cases} 0 := \emptyset \\ 1 := \emptyset^+ \\ 2 := \emptyset^{++} \\ \quad . \\ \quad . \\ \quad . \end{cases} \tag{4}$$

The concept comes from an unreleased paper by Zermelo-Fraenkel in 1916, as per the research.

Axiom 3. Each non-empty set *X* comprises a component $\propto$ which has no items in common with *X*, according to the Axiom of Regularity (REG) (IMAMURA, n.d.).

$$\forall X \neq \emptyset \exists \alpha(\alpha \in X \wedge \forall \beta(\beta \in \alpha \Rightarrow \beta \notin X)) \tag{5}$$

Definition 2. The two-tuple $\langle \propto, \beta \rangle$ is the paired collection of $\propto$ and the pair set of $\propto$ and β for certain objects which are $\propto$ and β; using the iota-operator, we obtain

$$\langle \propto, \boldsymbol{\beta} \rangle \coloneqq \boldsymbol{ix}(\forall \boldsymbol{\gamma}(\boldsymbol{\gamma} \in \boldsymbol{x} \Leftrightarrow \boldsymbol{\gamma} = \boldsymbol{\alpha} \vee \exists \boldsymbol{Z}(\boldsymbol{\gamma} = \boldsymbol{Z} \wedge \forall \boldsymbol{\eta}(\boldsymbol{\eta} \in \boldsymbol{Z} \Leftrightarrow \boldsymbol{\eta} = \propto \vee \boldsymbol{\eta} = \boldsymbol{\beta})))) \tag{6}$$

Definition 2's straightforward consequence is that the two-tuple $\langle \propto, \beta \rangle$exists for all items that are $\propto$ andβ. As a result, there is no risk of using nonsensical phrases while using the iota-operator in Definition 2.

Remark 3. The two-tuple $\langle \propto, \beta \rangle$could be recognized with the individuals $\{\propto, \{\propto, \beta\}\}$given individual objects $\propto$ and β in the world of the proposed design using set-builder notation.

These set-theoretical axioms are sufficient in this theory; nevertheless, the function-theoretical axioms in the following sections provide additional techniques for the building of sets.

Axioms 4. The Sum Set Axiom (SUM) states that for any set *X*, there exists a set *Y* made up of the components of *X*.

$$\forall X \exists Y \forall \alpha(\alpha \in Y \Leftrightarrow \exists Z(Z \in X \wedge \propto \in Z)) \tag{7}$$

Remark 4. Considering an individual's set $\underline{X}$ in the world of the proposed theory, the total set of $\underline{X}$ can be indicated by the character U $\underline{X}$ and recognized with the particular $\{\propto | \exists Z \in X(\alpha \in Z)\}$ using set-builder notation.

Axioms 5. The unlimited ordinal ω is the set of every limited Zermelo-Fraenkel ordinals, according to the Infinite Ordinal Axiom (INF).

$$0 \in \omega \wedge \forall \alpha(\alpha \in \omega \Rightarrow \propto^{+} \in \omega) \wedge \forall \beta \in \omega(Д\gamma \in \omega(\beta = \gamma^{+}) \Leftrightarrow \beta = \emptyset)) \tag{8}$$

Remark 5. With INF, the sets ω is defined in a unique way. The set ω could be expressed by the character $\mathbb{N}$ and recognised with the individual $\mathbb{N} \coloneqq \{\{\ \}, \{\{\ \}\}, \{\{\{\ \}\}\}, \ldots\}$ in the proposed architecture.

Axioms 6. Each functional $f_x$for every set *X* doesn't really receive anything within *X* as an argument:

$$\forall X \forall f_x \forall \alpha \notin X \forall \beta(f_x : \alpha \nmapsto \beta) \tag{9}$$

Remark 6. The previous Axiom states that for any$\propto \in X$, only single expression $f_x : \propto \rightarrow \beta$ is true. This would not rule out the possibility that such an expression is valid for something $\propto$other than *X*. This is, therefore, prohibited by Axiom 6.

Standard functional-Theory Axioms are as follows:

Axioms 7. There are no elements in a function f_x on a set *X*:

$$\forall X \forall f_X \forall \alpha(\alpha \notin f_X) \tag{10}$$

Remark 7. Some may believe that Axiom 7 negates the empty set's individuality. This, even though, is not the case. Although a functional on a set *X* and the empty set both have the characteristic of having no elements, the empty set is the unique set with this characteristic: Axiom 2 states unequivocally that a function on a set *X* is not really a sets.

Axioms 8. The Inverted Image Set Axiom (INV) states that for each non-empty sets *X* and every functional f_X with domains *X* and co-domains *Y*, there exists a set $Z \subset X$ that includes exactly the components of *X* transferred to a byf_X:

$$\forall X \neq \emptyset \forall f_X \forall Y(f_X : X \twoheadrightarrow Y \Rightarrow \forall \beta \exists Z \forall \alpha\ (\alpha \in Z \Leftrightarrow \propto \in X \wedge f_X : \propto \mapsto \beta)) \tag{11}$$

Remark 8. These axioms guarantees that the inverted imaging set of any item b in the co-domain of f_X obtains when we have generated an unique functional f_X with domain *X* from previous functionalities.

Remark 9. The original inverted imaging set could be represented by the character $f_X^{-1}(\beta)$ and could be recognised with the particular $(\propto \mid \propto \in X \wedge f_X : \propto \mapsto \beta\}$ in the world of the desired method, given an individual object β, individual set $\underline{X}$ and an individual function f_X, and an in the world of the desired prototype.

Axioms 9. Every functional f_X has a set *Y* as domain and a set *Z* as co-domain, and translates each component an in *Y* to an individual pictureβ: General Function-Theoretical Axiom (GEN-F):

$$\forall X \forall f_X(X \neq \emptyset \Rightarrow \exists Y \exists Z(f_X : Y \twoheadrightarrow Z \wedge \forall \alpha \in Y \exists ! \beta(f_X : \propto \mapsto \beta))) \tag{12}$$

Axioms 10. The Ur-Function Axiom (UFA) states that there occurs an ur-function $f_{\propto+}$ having domain $\propto^+$ and codomain β^+ that translates $\propto$ to β for every two items $\propto$ and β:

$$\forall\alpha\forall\beta\exists f_{\propto+}(f_{\propto+}:\propto^+ \twoheadrightarrow \beta^+ \wedge f_{\propto+}:\propto \mapsto \beta) \tag{13}$$

Remark 10. The ur-function $\{\propto\}$ on fag that translates $\propto$ to βcould be recognized with the individuals $\{\langle\propto,\beta\rangle\}_{\{\propto\}}$in the world of the desired prototype utilizing set-builder notation, provided unique items $\propto$ and β in the world of the desired form. It's worth noting that the graphing of the ur-function is certain to occur.

Core Theorems

The Graph Theorem states that for every set *X* and any function f_X with any co-domain *Y*, there exists a set *Z* whose members were the two-tuples $\langle\propto,\beta\rangle$ constructed by arguments and imaging of the functional f_X. In equation pattern:

$$\forall X\forall f_X\forall Y(f_X{:}X \twoheadrightarrow Y \Rightarrow \exists Z\forall\zeta\big(\zeta \in Z \Leftrightarrow \exists\alpha\exists\beta(\zeta = \langle\propto,\beta\rangle \wedge f_X{:}\propto \mapsto \beta)\big))) \tag{14}$$

Proof: Allow Ẋ to be any set, and the functional f_X to be any function on *X*. Because of GEN-F (Axiom 9), there is exactly one β for all $\propto\in X$ such that $f_X{:}\propto\mapsto\beta$.Applying Definition 8, there occurs a singleton $\langle\propto,\beta\rangle^+$ for each $\propto\in X$ such that$f_X{:}\propto\mapsto\beta$. There is also an ur-function $u_{\propto+}{:}\propto^+\twoheadrightarrow\langle\propto,\beta\rangle^+$, $u_{\propto+}{:}\propto\mapsto\langle\propto,\beta\rangle$ for each $\propto\in X$ because of the ur-function axiom (Axiom 10) (Herman and Caughman 2021) (Rathjen 2005). As a result of SUM-F, there exists a sum function G_X with a co-domain *Z* such that G_X translates any $\propto\in X$ to the two-tuple $\langle\propto,\beta\rangle$ for which $f_X{:}\propto\mapsto\beta$.The co-domain *Z* of G_X occurs because of GEN-F, and it is peculiar because of Axiom 7:

There is no risk in associating a functional *f* with the singleton graph $(f)_{dom(f)}$, where graph *(f)* is the graph of *f* and dom(*f*) is the domain of *f*, *cf* in ζ's proposed framework.

Theorem 2. (Main theorem) When there is a functional correlation $\phi(\propto,\beta)$ that connects every $\propto$ in *X* to exactly oneβ, and then there is a functional f_X with certain co-domain *Y* that connects each $\eta\in X$ to exactly that $\zeta\in Y$for which exists $\phi(\eta,\zeta)$.Using the iota-operator in an equation:

$$\forall X\neq\emptyset(\forall\alpha\in X\exists!\,\beta\phi(\propto,\beta)\Rightarrow\exists F_X\exists Y\Big(F_X{:}X\twoheadrightarrow Y\forall\eta\in X\big(F_X{:}\eta\mapsto I\zeta\phi(\eta,\zeta)\big)\Big)) \tag{15}$$

Proof: Let *X* be a non - empty set of any size. Assume that for each$\propto\in X$, there is exactly one β such that$\phi(\propto,\beta)$. The ur-function axiom (Axiom 18) states that for any arbitrary consistent$\propto\in X$, there occurs always an ur-function u_{a+}.

$$u_{a+}{:}\propto\mapsto \boldsymbol{i\beta}\phi(\propto,\boldsymbol{\beta}) \tag{16}$$

Let $f_{\propto+}$ be a parameter that spans these ur-functions. Then, using Numerous International Elimination and then Non-standard International Elimination, we derive from SUM-F that

$$\exists F_X\exists Y(F_X{:}X\twoheadrightarrow Y\wedge\wedge_{\propto\in X}F_X{:}\alpha\mapsto i\beta(f_{\alpha+}{:}\alpha\mapsto\beta)) \tag{17}$$

We next infer the framework by implementing Conjunctive Operator Elimination and Rule-C.

$$F_X{:}\alpha\mapsto\boldsymbol{i\beta}\phi(\propto,\boldsymbol{\beta}) \tag{18}$$

For the f_X sum function. We get the following when we generalise this model.

$$\forall\eta\in X(F_X{:}\eta\mapsto I\zeta\phi(\eta,\zeta))) \tag{19}$$

As a result,

$$\exists F_X\exists Y(F_X{:}X\twoheadrightarrow Y\wedge\forall\eta\in X\big(F_X{:}\eta\mapsto i\zeta\phi(\eta,\zeta)\big))) \tag{20}$$

Derivation of SEP and REP of Zermelo-Fraenkel

We begin by demonstrating that Zermelo-Fraenkel's limitless axiom structure SEP is a theorem (template) of ϰ:

Theorem 3. ZF's Separation Axiom Framework:

$$\forall X\exists Y\forall\alpha(\alpha\in Y\Leftrightarrow\propto\in X\wedge\phi(\propto)) \tag{15}$$

Proof: Let ϕ be an arbitrary unary connection on *X* and *X* is an arbitrary set. According to the ur-function axiom (Axiom 18), there occurs an ur-function $u_{\propto+}$ for any arbitrary constants$\propto\in X$.

$$\begin{cases} u_{\propto^+;\propto\mapsto 1\ if\ \phi(\propto)} \\ u_{\propto^+;\propto\mapsto 0\ if\ \phi(\propto)} \end{cases} \tag{16}$$

Let $f_{\eta+}$ be a variable that spans these ur-functions. As a result of Theorem 2, we obtain

$$\exists F_X \forall \eta \in X(F_X{:}\,\eta \mapsto i\beta(f_{\eta+}{:}\,\eta \mapsto \beta)) \tag{17}$$

The constant f_X will be used to represent this sum function. The inverted imaging set $F_X^{-1}(1)$ occurs due to INV (Axiom 8): we obtain $\forall\alpha(\alpha \in F_X^{-1}(1) \Leftrightarrow\propto\in X\wedge\phi(\propto))$. Theorem 3 follows from here, thanks to global and existential generalisation.

The unlimited axiom structure REP of Zermelo-Fraenkel is therefore proved to be a theorem (template) of our concept א:

Theorem 4. Zermelo-Fraenkel's Replacement Axiom Framework:

$$\forall X(\forall\alpha \in X\exists!\,\beta\phi(\propto,\beta) \Rightarrow \exists Z\forall\gamma\Big(\gamma \in Z \Leftrightarrow \exists\zeta\big(\zeta \in X\wedge\phi(\zeta,\gamma)\big)\Big)) \tag{18}$$

Proof: Let *X* be an arbitrary set, so that there can be exactly one β for each $\propto\in X$ so that$\phi(\propto,\beta)$. Therefore, due to Theorem 2, there occurs a sum functional f_X for which

$$\forall\alpha \in X(F_X{:}\,\alpha \mapsto i\beta\phi(\propto,\beta)) \tag{19}$$

Because of Axiom 6, the imaging set is the co-domain ofF_X; indicating this with$F_X[X]$, we obtain

$$\forall\gamma(\gamma \in F_X[X] \Leftrightarrow \exists\zeta\big(\zeta \in X \wedge \phi(\zeta,\gamma)\big)) \tag{20}$$

This is suggested by $\forall\alpha \in X\exists!\,\beta\phi(\propto,\beta)$ because the functioning relation ϕ on the arbitrary set *X* was established$(\propto,\beta)$. We publish down the following inference: Theorem 4 is thus obtained through global and existential generalisation.

The structure of these two theorems offers evidence that our theory א is not lower than Zermelo-Fraenkel (Cabbolet and de Swart 2014b). Furthermore, we have not yet proved that each Zermelo-Fraenkel result regarding sets instantly transfers to the current structure, in which not everything is a set.

Conclusion

The key result is that the goal mentioned in the introductory has been met: a theory א has been created with a vocabulary including a finitely large number of constants that excludes the 2 undesirable pathological properties of Zermelo-Fraenkel. Because each of א's axioms is a typographically limited statement, unlike infinitely axioms, each axiom may be stated out directly. Not only that, but unlike ZF, א is finitely axiomatized, which means that the complete theory א may be put out directly on a sheet of papers. Furthermore, it has been demonstrated that, unlike ZF, א doesn't even have a calculable framework it has one at all. The non-classical structure of א accounts for the defeat of the descending Löwenheim–Skolem theorem for א. Moreover, there are 3 ways why א could be used as a basic theory in mathematical theorem. Firstly and foremost, it has been demonstrated that the axioms of Zermelo-Fraenkel may be deduced from א. Whereas we recognise that this conclusion doesn't really indicate that those axioms regarding sets obtained in the structure of ZF are essentially accurate in the structure of א, since sets with non-set components occur in the latter structure, it does demonstrate that the techniques for creating sets accessible in the structure of ZF are also existing in the schema of א. Consequently, the axioms of categories have been demonstrated to satisfy for the world of discussion connected with א, which is a categorization of functions and sets: this world may also provide as the ontological foundation for the many (big) categories investigated in categorical theories. Finally, א is simple to use in daily mathematics because we could create a functional F_X for any set *X* by offering a determining features $F_X{:}\,X \twoheadrightarrow Y, F_X{:}\propto\overset{def}{\longrightarrow}\imath\beta\phi(\propto,\beta)$ in which ϕ is some functional connection: א then assures thatF_X, as well as its invertible portrait, graph, and image sets for each component of its co-domain, exists—ergo, giving a determining function for creating sets on prescription. All of these approaches to a solution have remarks, and the writers aren't sure which is the finest. The goal of this study was to get these concerns more broadly acknowledged, particularly among mathematicians who do not research in set theory. We believe that by performing so, we will spark a lively debate on the contradictions that exist among theoretical set of axioms, and their infinite sets, as well as the Axiom of Choice.

References

Abd Algani, Y. (2025). A hybrid framework to improve hostel security using MTCNN, GDHASLR and BPO. International Journal of Information Technology -Springer. https://doi.org/10.1007/s41870-025-02769-6

Abd Algani, Y.M. (2024). A Novel Deep Learning Attention Based Sequence to Sequence Model for Automatic Abstractive Text Summarization. International Journal of Information Technology -Springer.

Abd Algani, Y.M. (2023). Resource Allocation in IoT-based Edge Computing using Deep Learning with Optimization Model," 2023 Second International Conference on Electronics and Renewable Systems (ICEARS), IEEE, pp. 516-522

Abd Algani, Y., Amal Sharif- Rasslan. The Topological Connectivity of the Bipartite Graph's Independence Complex. In Paul Bracken. (2023) Topology - Recent Advances and Applications. (Chapter 1) Intechopen, UK.

Abd Algani, Y. (2021). Multimodal Interaction Using the Particle SWARM'S Binary Optimization. In Kumar, N., Gelman, L., Bar, A.K., & Chakrabarti, S. (Eds.): (2021). Intelligent and Reliable Engineering Systems: 11th International Conference on Intelligent Energy Management, Electronics, Electric & Thermal Power, Robotics and Automation (IEMERA-2020) (1st ed.). (Chapter 28 180- 189).

Antos, Carolin, Daniel Kuby, Neil Barton, and Deborah Kant. n.d. "Set Theory: Bridging Mathematics and Philosophy."

Assaf, Ali, Guillaume Burel, Raphal Cauderlier, David Delahaye, Gilles Dowek, Catherine Dubois, Frédéric Gilbert, Pierre Halmagrand, Olivier Hermant, and Ronan Saillard (2016). "Expressing Theories in the ΛΠ-Calculus modulo Theory and in the Dedukti System." In 22nd International Conference on Types for Proofs and Programs.

Bendkowski, Maciej (2020). "A Note on the Asymptotic Expressiveness of ZF and ZFC." ArXiv Preprint ArXiv:2010.10155.

Bingham, Nicholas H, and Adam J Ostaszewski (2019). "Set Theory and the Analyst." European Journal of Mathematics 5 (1): 2–48.

Boulabiar, Karim (2015). "Lattice and Algebra Homomorphisms on C (X) in Zermelo-Fraenkel Set Theory." Quaestiones Mathematicae 38 (6): 835–39.

Cabbolet, Marcoen JTF (2021). "A Finitely Axiomatized Non-Classical First-Order Theory Incorporating Category Theory and Axiomatic Set Theory." Axioms 10 (2): 119.

Cabbolet, Marcoen JTF, and Harrie CM de Swart (2014a). "Set Matrix Theory as a Physically Motivated Generalization of Zermelo-Fraenkel Set Theory." Logique et Analyse, 59–82.

Cabbolet and de Swart (2014b). "Set Matrix Theory as a Physically Motivated Generalization of Zermelo-Fraenkel Set Theory." Logique et Analyse, 59–82.

Cavitt, Justin William Henry. 2017. "Set-Theoretic Geology, the Ultimate Inner Model, and New Axioms." PhD Thesis, Harvard University.

Christopoulou, Demetra. 2020. "Implicitly Defining Mathematical Terms: A Path Toward Pluralism." Handbook of the History and Philosophy of Mathematical Practice, 1–17.

Crosilla, Laura. 2019. "The Entanglement of Logic and Set Theory, Constructively." Inquiry, 1–22.

Dunne, Ciarán, Joseph Brian Wells, and Fairouz Kamareddine. 2020a. "Adding an Abstraction Barrier to ZF Set Theory." In International Conference on Intelligent Computer Mathematics, 89–104. Springer.

Dunne, Wells, and Kamareddine (2020b). "Adding an Abstraction Barrier to ZF Set Theory." In International Conference on Intelligent Computer Mathematics, 89–104. Springer.

Erkan, Ekin (2020). "For a Rationalist Politics of the Event: Zermelo–Fraenkel Set Theory and Structuring the Multiple." Filozofski Vestnik 41 (1).

Fernengel, Anne, and Peter Koepke (2016). "An Easton-like Theorem for Zermelo-Fraenkel Set Theory without Choice." ArXiv Preprint ArXiv:1607.00205.

Freiman, Robert (2020). "Comparison and Analysis of Constructive Set Theories." PhD Thesis, Wien.

FREIRE, ALFREDO R. n.d. "COMPARING THE STRENGTH OF ONTOLOGICAL COMMITMENT BETWEEN THEORIES."

Garcia Tarrach, Gina (2017). "The Axiom of Choice and Its Implications in Mathematics."

Herman, Ari, and John Caughman (2021). "Probability Axioms and Set Theory Paradoxes." Symmetry 13 (2): 179.

Herzberg, Frederik (2016). "The Transfer Principle Holds for Definable Nonstandard Models under Countable Choice." Center for Mathematical Economics Working Papers.

IMAMURA, TAKUMA. n.d. "USEFULNESS OF AXIOMS AND SPEEDUP THEOREM."

Jia, Ruoxuan (2017). "The Axiom of Choice in Topology."

Makarov, Victor (2021). "An Axiomatic Definition of Ordered Pairs in Zermelo-Fraenkel Set Theory."

Passmann, Robert (2019a). “De Jongh’s Theorem for Intuitionistic Zermelo-Fraenkel Set Theory.” ArXiv Preprint ArXiv:1905.04972.

Passmann (2019b). “The de Jongh Property for Bounded Constructive Zermelo-Fraenkel Set Theory.” ILLC Prepublication Series PP-2019-05.

Paulson, Lawrence C. 2019. “Zermelo Fraenkel Set Theory in Higher-Order Logic.” Archive of Formal Proofs, 91.

Rathjen, Michael. 2005. “Replacement versus Collection and Related Topics in Constructive Zermelo–Fraenkel Set Theory.” Annals of Pure and Applied Logic 136 (1–2): 156–74.

Shin, Kevin. 2019. “The Independence of the Continuum Hypothesis in Zermelo-Fraenkel Set Theory.”

Sieg, Wilfried, and Patrick Walsh. 2021. “Natural Formalization: Deriving the Cantor-Bernstein Theorem in ZF.” The Review of Symbolic Logic 14 (1): 250–84.

Wehr, Dominik. 2020. “Aczel’s Type-Theoretic Interpretation of Constructive Zermelo-Fraenkel Set Theory.”

Zakharov, Valeriy K, and Timofey V Rodionov. 2018. “A Characterization of All Natural Models of Neumann–Bernays–Gödel and Zermelo–Fraenkel Set Theories.” In Fundamentals of Set and Number Theory, 185–280. De Gruyter.

Kumaraswamy-Switch-Time distributions

Pavel Stoynov
Perun R&D Academy,
Institute of Information Sciences,
Sofia, Bulgaria
pavel_todorov_stoynov@yahoo.com

Abstract.
In this article we consider Kumaraswamy-G distribution and take G to be switch-time (ST) distribution. Among the obtainet Kumaraswamy-Switch-Time distributions, we consider the special cases of Kumaraswamy-exponential, Kumaraswamy-Lindley and Kumaraswamy-Stoynov distribution.
Keywords: finance, financial group, funding, business plan,

Introduction

The classical Kumaraswamy distribution (Kumaraswamy, 1980) is the distribution with cumulative distribution function

$$F_K(x;\alpha,\beta)=1-(1-x^{\alpha})^{\beta},\ x\in(0,1),\ \alpha>0,\ \beta>0 \tag{1}$$

and probability density function

$$f_K(x;\alpha,\beta)=\alpha\beta x^{\alpha-1}(1-x^{\alpha})^{\beta-1},\ x\in(0,1),\ \alpha>0,\ \beta>0. \tag{2}$$

For any baseline cumulative distribution function $G(x)$, Cordeiro and Castro (2011) proposed Kumarswamy-G distribution with cumulative distribution function

$$F_{KG}(x;\alpha,\beta)=1-(1-G^{\alpha}(x))^{\beta},\ \alpha>0,\ \beta>0 \tag{3}$$

and probability density function

$$f_{KG}(x;\alpha,\beta)=\alpha\beta g(x)G^{\alpha-1}(x)(1-G^{\alpha}(x))^{\beta-1},\quad \alpha>0,\ \beta>0,\ g(x)=\frac{dG(x)}{dx}. \tag{4}$$

In this article, we choose $G(x)$ to be swich-time (ST) distribution and more precisely, we consider three special cases of switch-time: exponential, Lindley and Stoynov distributions. The corresponding Kumaraswamy-G distributions are: Kumaraswamy-exponential, Kumaraswamy-Lindley and Kumaraswamy-Stoynov distribution.

ST distribution

A random variable ξ with a switching distribution ST(n,ß) (Stoynov, 2026) has a density

$$f_{\xi}(x)=\begin{cases}C(n,\beta)e^{-\beta x}(1+x)^n, & x\geq 0\\ 0, & x<0,\end{cases} \tag{5}$$

where $C(n,\beta)$ are normalization coefficients for which

$$C(n,\beta)=\frac{1}{I(n,\beta)} \tag{6}$$

and

$$I(n,\beta)=\frac{1}{\Gamma(1)}\int_0^{\infty}e^{-\beta t}(1+t)^n\,dt=\int_0^{\infty}e^{-\beta t}(1+t)^n\,dt. \tag{7}$$

In private cases when $n=0,\ n=1$ and $n=2$ the equalities $C(0,\beta)=\beta,\ C(1,\beta)=\frac{\beta^2}{\beta+1}$ and $C(2,\beta)=\frac{\beta^3}{\beta^2+2\beta+2}$ apply accordingly. The first and second case are exponential and Lindley

distribution correspondingly. The third case is sometimes also called Stoynov distribution. And in the case when $\beta = 1$ we have correspondingly standard exponential, standard Lindley and strandard Stoynov distribution.

A random variable ξ with exponential distribution has density

$$f_\xi(x) = \begin{cases} \beta e^{-\beta x} & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{8}$$

The corresponding cumulative distribution function is

$$F_\xi(x) = \begin{cases} 1 - e^{-\beta x} & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{9}$$

A random variable ξ with standard exponential distribution has density

$$f_\xi(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{9}$$

The corresponding cumulative distribution function of standard exponential is

$$F_\xi(x) = \begin{cases} 1 - e^{-x} & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{10}$$

A random variable ξ with Lindley distribution has density

$$f_\xi(x) = \begin{cases} \dfrac{\beta^2}{\beta + 1} e^{-\beta x}(1 + x), & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{11}$$

The corresponding cumulative distribution function is

$$F_\xi(x) = \begin{cases} 1 - e^{-\beta x} - \dfrac{\beta}{\beta + 1} x e^{-\beta x} & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{12}$$

A random variable ξ with standard Lindley distribution has density

$$f_\xi(x) = \begin{cases} \dfrac{1}{2} e^{-x}(1 + x), & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{13}$$

The corresponding cumulative distribution function is

$$F_\xi(x) = \begin{cases} 1 - e^{-x} - \dfrac{1}{2} x e^{-x} & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{14}$$

The third case is ST(2,ß) or Stoynov distribution with probability density

$$f_\xi(x) = \begin{cases} \dfrac{\beta^3}{\beta^2 + 2\beta + 2} e^{-\beta x}(1 + x)^2, & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{15}$$

The corresponding cumulative distribution function is

$$F_\xi(x) = \begin{cases} \dfrac{\beta}{\beta^2 + 2\beta + 2} \times \\ \times (\beta + 4 - (\beta + 4)e^{-\beta x} - 2(\beta + 1)x e^{-\beta x} - \beta x^2 e^{-\beta x}), \\ x \geq 0, \\ 0, x < 0. \end{cases} \tag{16}$$

This cumulative distribution function can also be presented as

$$F_\xi(x) = \begin{cases} \dfrac{\beta}{\beta^2 + 2\beta + 2} \times \\ \times (\beta + 4 - 2e^{-\beta x} - 2(1 + x)e^{-\beta x} - \beta(1 + x)^2 e^{-\beta x}), \\ x \geq 0, \\ 0, x < 0. \end{cases} \tag{17}$$

When the second parameter is $\beta = 1,$ the density is

$$f_{\xi}(x)=\begin{cases}\frac{1}{5}e^{-x}(1+x)^2,\ x\geq 0\\ 0,\ x<0.\end{cases} \tag{18}$$

The cumulative distribution function for ST(2,1) distribution (standard Stoynovdistribution) can be derived as

$$F_{\xi}(x)=\begin{cases}1-e^{-x}-\frac{4}{5}xe^{-x}-\frac{1}{5}x^2e^{-x},\ x\geq 0\\ 0,\ x<0.\end{cases} \tag{19}$$

In the next part, we combine these distribution with Kumaraswamy distribution to obtain special cases of Kumaraswamy-G distribution.

Kumaraswamy-ST distributions

Considering Kumaraswamy-G distribution where $G\equiv ST(n,\beta),$ we obtain diifeferent types of Kumaraswamy-ST distributions. We consider here several special cases of Kumaraswamy-ST distributions.

In this way we will have correspondingly:

1. Kumaraswamy-Exponential distribution or Kumaraswamy-$ST(0,\theta)$,with cumulative distribution function

$$F_{KE}(x;\alpha,\beta,\theta)=\begin{cases}1-(1-(1-e^{-\theta x})^{\alpha})^{\beta},\\ x\geq 0;\ \alpha>0,\ \beta>0,\theta>0\\ 0,\ x<0\end{cases} \tag{20}$$

and probability density function

$$f_{KE}(x;\alpha,\beta,\theta)=\begin{cases}\alpha\beta\theta e^{-\theta x}(1-e^{-\theta x})^{\alpha-1}(1-(1-e^{-\theta x})^{\alpha})^{\beta-1},\\ x\geq 0;\ \alpha>0,\ \beta>0,\theta>0\\ 0,\ x<0\end{cases} \tag{21}$$

In the case when $\theta=1,$ we obtain Kumaraswamy-Standard Exponential distribution with cumulative distribution function

$$F_{KE}(x;\alpha,\beta)=\begin{cases}1-(1-(1-e^{-x})^{\alpha})^{\beta},\ x\geq 0;\ \alpha>0,\ \beta>0\\ 0,\ x<0\end{cases} \tag{22}$$

and probability density function

$$f_{KE}(x;\alpha,\beta)=\begin{cases}\alpha\beta e^{-x}(1-e^{-x})^{\alpha-1}(1-(1-e^{-x})^{\alpha})^{\beta-1},\\ x\geq 0;\ \alpha>0,\ \beta>0\\ 0,\ x<0\end{cases} \tag{23}$$

2. Kumaraswamy-Lindley distribution or Kumaraswany-$ST(1,\theta)$ distribution has cumulative distribution function

$$F_{KL}(x;\alpha,\beta,\theta)=\begin{cases}1-(1-(1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x})^{\alpha})^{\beta},\\ x\geq 0;\ \alpha>0,\ \beta>0\\ 0,\ x<0\end{cases} \tag{24}$$

and probability density function

$$f_{KL}(x;\alpha,\beta,\theta)=\begin{cases}\alpha\beta\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}(1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x})^{\alpha-1}\times\\ \times(1-(1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x})^{\alpha})^{\beta-1},\\ x\geq 0;\ \alpha>0,\ \beta>0\\ 0,\ x<0\end{cases} \tag{25}$$

In the case when $\theta=1,$ we obtain Kumaraswamy-Standard Lindley distribution with cumulative distribution function

$$F_{KL}(x;\alpha,\beta)=\begin{cases}1-(1-(1-\frac{2+x}{2}e^{-x})^{\alpha})^{\beta}, x\geq 0;\alpha>0,\beta>0\\ 0, x<0\end{cases} \tag{26}$$

and probability density function

$$f_{KL}(x;\alpha,\beta)=\begin{cases}\alpha\beta\frac{1}{2}(1+x)e^{-x}(1-\frac{2+x}{2}e^{-x})^{\alpha-1}\times\\ \times(1-(1-\frac{2+x}{2}e^{-x})^{\alpha})^{\beta-1}, x\geq 0;\alpha>0,\beta>0\\ 0, x<0\end{cases} \tag{27}$$

3. Kumaraswamy-Stoynov or Kumaraswany- $ST(2,1)$ distribution has cumulative distribution function

$$F_{KS}(x;\alpha,\beta,\theta)=\begin{cases}1-(1-(\frac{\theta}{\theta^2+2\theta+2}\times\\ \times(\theta+4-2e^{-\theta x}-2(1+x)e^{-\theta x}-\\ -\theta(1+x)^2e^{-\theta x})^{\alpha})^{\beta},\\ x\geq 0;\alpha>0,\beta>0\\ 0, x<0\end{cases} \tag{28}$$

and probability density function

$$f_{KS}(x;\alpha,\beta,\theta)=\begin{cases}\alpha\beta\frac{\theta^3}{\theta^2+2\theta+1}(1+x)^2e^{-\theta x}\times\\ \times(1-\frac{\theta}{\theta^2+2\theta+2}(\theta+4-2e^{-\theta x}-2(1+x)e^{-\theta x}-\\ -\theta(1+x)^2e^{-\theta x}))^{\alpha-1}\times\\ \times(1-(1-\frac{\theta}{\theta^2+2\theta+2}\times\\ \times(\theta+4-2e^{-\theta x}-2(1+x)e^{-\theta x}-\\ -\theta(1+x)^2e^{-\theta x})^{\alpha})^{\beta-1},\\ x\geq 0;\alpha>0,\beta>0\\ 0, x<0\end{cases} \tag{29}$$

In the case when $\theta=1,$ we obtain Kumaraswamy-Standard Stoynov distribution with cumulative distribution function

$$F_{KS}(x;\alpha,\beta)=\begin{cases}1-(1-(1-e^{-x}-\frac{4}{5}xe^{-x}-\frac{1}{5}x^2e^{-x})^{\alpha})^{\beta},\\ x\geq 0;\alpha>0,\beta>0\\ 0, x<0\end{cases} \tag{30}$$

and probability density function

$$f_{KS}(x;\alpha,\beta)=\begin{cases}\alpha\beta\frac{1}{5}(1+x)^2e^{-x}\times\\ (1-e^{-x}-\frac{4}{5}xe^{-x}-\frac{1}{5}x^2e^{-x})^{\alpha-1}\times\\ \times(1-(1-e^{-x}-\frac{4}{5}xe^{-x}-\frac{1}{5}x^2e^{-x})^{\alpha})^{\beta-1},\\ x\geq 0;\alpha>0,\beta>0\\ 0, x<0\end{cases} \tag{31}$$

If we chose $\alpha = \beta = 1$, Kumaraswamy-G distribution becomes just G distribution, so, in our case, if we select the parameters in that way, we obtain the corresponding ST distribution – exponential, Lindley or Stoynov.

Conclusion

Kumaraswamy-ST as a special case of Kumaraswamy-G distributions could have many application in mathematics, IT, natural and social sciences. In a next article on this topic, the author plans to present some applications as well as some properties of this class of distributions.

References

Cordeiro, G. M. and de Castro, M. (2011) A new family of generalized distributions. Journal of Statistical Computation and Simulation, 81, 883-893. https://doi.org/10.1080/00949650903530745

Kumaraswamy, P. (1980) A generalized probability density function for double-bounded random processes. Journal of Hydrology, 46(1-2), 79-88.

Stoynov, P. (2016) Switch Time Family of distributions and processes and their applications to reflected surplus models. Annual of the Faculty of Economics and Business Administration, Sofia University "St. Kliment Ohridski" - Sofia, 2016, pp. 255-285.